

V - A hadronic tau decays : a QCD laboratory *

Stephan Narison^a

^aLaboratoire de Physique Mathématique et Théorique, Université de Montpellier II Place Eugène Bataillon, 34095 - Montpellier Cedex 05, France
E-mail: narison@lpm.univ-montp2.fr

Recent ALEPH/OPAL data on the V - A spectral functions from hadronic τ decays are used for fixing the QCD continuum threshold at which the first and second Weinberg sum rules should be satisfied in the chiral limit, and for predicting the values of the low-energy constants f_π , $m_{\pi^+} - m_{\pi^0}$ and L_{10} . Some DMO-like sum rules and the τ -total hadronic widths $R_{\tau,V-A}$ are also used for extracting the values of the $D = 6, 8$ QCD vacuum condensates and the corresponding (in the chiral limit) electroweak kaon penguin matrix elements $\langle Q_{8,7}^{3/2} \rangle_{2\pi}$, where a deviation from the vacuum saturation estimate has been obtained. Combining these results with the one of the QCD penguin matrix element $\langle Q_6^{1/2} \rangle_{2\pi}$ obtained from a (maximal) $\bar{q}q$ -gluonium mixing scheme from the scalar meson sum rules, we deduce, in the Electroweak Standard Model (ESM), the conservative upper bound for the CP-violating ratio: $\epsilon'/\epsilon \leq (22 \pm 9)10^{-4}$, in agreement with the present measurements.

1. Introduction

Hadronic tau decays have been demonstrated [1] (hereafter referred as BNP) to be an efficient laboratory for testing perturbative and non-perturbative QCD. That is due both to the exceptional value of the tau mass situated at a frontier regime between perturbative and non-perturbative QCD and to the excellent quality of the ALEPH/OPAL [2,3] data. On the other, it is also known before the advent of QCD, that the Weinberg [4] and DMO [5] sum rules are important tools for controlling the chiral and flavour symmetry realizations of QCD, which are broken by light quark mass terms to higher order [6] and by higher dimensions QCD condensates [7] within the SVZ expansion [8].

In this talk, we shall discuss the impact of the new ALEPH/OPAL data on the V - A spectral functions in the analysis of the previous and some other related sum rules, which will be used for determining the low-energy constants of the effective chiral lagrangian [9–11], the SVZ QCD vacuum condensates [8]. In particular, we shall discuss the consequences of these results on the

estimate of the kaon CP -violation parameter ϵ'/ϵ in the Electroweak Standard Model (ESM). These results have been originally obtained in [12] and will be reviewed here.

2. Tests of the “sacrosante” Weinberg and DMO sum rules in the chiral limit

2.1. Notations

We shall be concerned here with the two-point correlator:

$$\begin{aligned} \Pi_{LR}^{\mu\nu}(q) &\equiv i \int d^4x e^{iqx} \langle 0 | T J_L^\mu(x) (J_R^\nu(0))^\dagger | 0 \rangle \\ &= -(g^{\mu\nu} q^2 - q^\mu q^\nu) \Pi_{LR}(q^2), \end{aligned} \quad (1)$$

built from the left- and right-handed components of the local weak current:

$$J_L^\mu = \bar{u} \gamma^\mu (1 - \gamma_5) d, \quad J_R^\mu = \bar{u} \gamma^\mu (1 + \gamma_5) d, \quad (2)$$

and/or using isospin rotation relating the neutral and charged weak currents:

$$\rho_V - \rho_A \equiv \frac{1}{2\pi} \text{Im} \Pi_{LR} \equiv \frac{1}{4\pi^2} (v - a). \quad (3)$$

The first term is the notation in [13], while the last one is the notation in [2,3].

*Talk given at the QCD 00 International Euroconference (Montpellier 6-13th July 2000). This is a summary of the paper “New QCD Estimate of the Kaon Penguin Matrix Elements” hep-ph/0004247 (Nucl. Phys. B in press).

2.2. The sum rules

The “sacrosante” DMO and Weinberg sum rules read in the chiral limit ²:

$$\begin{aligned}
\mathcal{S}_0 &\equiv \int_0^\infty ds \frac{1}{2\pi} \text{Im}\Pi_{LR} = f_\pi^2 , \\
\mathcal{S}_1 &\equiv \int_0^\infty ds s \frac{1}{2\pi} \text{Im}\Pi_{LR} = 0 , \\
\mathcal{S}_{-1} &\equiv \int_0^\infty \frac{ds}{s} \frac{1}{2\pi} \text{Im}\Pi_{LR} = -4L_{10} , \\
\mathcal{S}_{em} &\equiv \int_0^\infty ds \left(s \log \frac{s}{\mu^2} \right) \frac{1}{2\pi} \text{Im}\Pi_{LR} \\
&= -\frac{4\pi}{3\alpha} f_\pi^2 (m_{\pi^\pm}^2 - m_{\pi^0}^2) , \tag{4}
\end{aligned}$$

where $f_\pi|_{exp} = (92.4 \pm 0.26)$ MeV is the experimental pion decay constant which should be used here as we shall use data from τ -decays involving physical pions; $m_{\pi^\pm} - m_{\pi^0}|_{exp} \simeq 4.5936(5)$ MeV; $L_{10} \equiv f_\pi^2 \langle r_\pi^2 \rangle / 3 - F_A$ [$\langle r_\pi^2 \rangle = (0.439 \pm 0.008) fm^2$ is the mean pion radius and $F_A = 0.0058 \pm 0.0008$ is the axial-vector pion form factor for $\pi \rightarrow e\nu\gamma$] is one the low-energy constants of the effective chiral lagrangian [9–11]. In order to exploit these sum rules using the ALEPH/OPAL [2,3] data from the hadronic tau-decays, we shall work with their Finite Energy Sum Rule (FESR) versions (see e.g. [6,1] for such a derivation). In the chiral limit ($m_q = 0$ and $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle$), this is equivalent to truncate the LHS at t_c until which the data are available, while the RHS of the integral remains valid to leading order in the $1/t_c$ expansion in the chiral limit, as the breaking of these sum rules by higher dimension $D = 6$ condensates in the chiral limit which is of the order of $1/t_c^3$ is numerically negligible [7].

2.3. Matching between the low and high-energy regions

In order to fix the t_c values which separate the low and high energy parts of the spectral functions, we require that the 2nd Weinberg sum rule (WSR) \mathcal{S}_1 should be satisfied by the present data. As shown in Fig. 1 (see [12]), this is obtained for two values of t_c ³:

$$t_c \simeq (1.4 \sim 1.5) \text{ GeV}^2 \quad \text{and} \quad (2.4 \sim 2.6) \text{ GeV}^2. \tag{5}$$

²Systematic analysis of the breaking of these sum rules by light quark masses [6] and condensates [7,8] within the context of QCD have been done earlier.

³One can compare the two solutions with the t_c -stability region around 2 GeV² in the QCD spectral sum rules analysis (see e.g. Chapter 6 of [14]).

Figure 1. FESR version of the 2nd Weinberg sum rule versus t_c in GeV² using the ALEPH/OPAL data of the spectral functions. Only the central values are shown.

Though the 2nd value is interesting from the point of view of the QCD perturbative calculations (better convergence of the QCD series), its exact value is strongly affected by the inaccuracy of the data near the τ -mass (with the low values of the ALEPH/OPAL data points, the 2nd Weinberg sum rule is only satisfied at the former value of t_c).

After having these t_c solutions, we can improve the constraints by requiring that the 1st Weinberg sum rule \mathcal{S}_0 reproduces the experimental value of f_π ⁴ within an accuracy 2-times the experimental error. This condition allows to fix t_c in a very narrow margin due to the sensitivity of the result on the changes of t_c values ⁵

$$t_c = (1.475 \pm 0.015) \text{ GeV}^2 , \tag{6}$$

3. Low-energy constants L_{10} , $m_{\pi^\pm} - m_{\pi^0}$ and f_π in the chiral limit

Using the previous value of t_c into the \mathcal{S}_{-1} sum rule, we deduce:

$$L_{10} \simeq -(6.26 \pm 0.04) \times 10^{-3} , \tag{7}$$

which agrees quite well with more involved analysis including chiral symmetry breakings [15,3], and with the one using a lowest meson dominance (LMD) of the spectral integral [16].

Analogously, one obtains from the \mathcal{S}_{em} sum rule:

$$\Delta m_\pi \equiv m_{\pi^\pm} - m_{\pi^0} \simeq (4.84 \pm 0.21) \text{ MeV} . \tag{8}$$

This result is 1σ higher than the data 4.5936(5) MeV, but agrees within the errors with the more detailed analysis from τ -decays [17,3] and with the LMD result of about 5 MeV [16]. We have checked that moving the subtraction point μ from 2 to 4 GeV slightly decreases the value of Δm_π by 3.7% which is relatively weak, as expected. Indeed, in the chiral limit, the μ dependence does not appear in the RHS of the \mathcal{S}_{em} sum rule, and then, it looks natural to choose:

$$\mu^2 = t_c , \tag{9}$$

⁴Though we are working here in the chiral limit, the data are obtained for physical pions, such that the corresponding value of f_π should also correspond to the experimental one.

⁵For the second set of t_c -values in Eq. 5, one obtains a slightly lower value: $f_\pi = (84.1 \pm 4.4)$ MeV.

because t_c is the only external scale in the analysis. At this scale the result increases slightly by 2.5%. One can also notice that the prediction for Δm is more stable when one changes the value of $t_c = \mu^2$. Therefore, the final predictions from the value of t_c in Eq. (6) fixed from the 1st and 2nd Weinberg sum rules are:

$$\begin{aligned}\Delta m &\simeq (4.96 \pm 0.22) \text{ MeV} , \\ L_{10} &\simeq -(6.42 \pm 0.04) \times 10^{-3} ,\end{aligned}\quad (10)$$

which we consider as our "best" predictions.

For some more conservative results, we also give the predictions obtained from the second t_c -value given in Eq. (5). In this way, one obtains:

$$\begin{aligned}f_\pi &= (87 \pm 4) \text{ MeV} , \\ \Delta m &\simeq (3.4 \pm 0.3) \text{ MeV} , \\ L_{10} &\simeq -(5.91 \pm 0.08) \times 10^{-3} ,\end{aligned}\quad (11)$$

where one can notice that the results are systematically lower than the ones obtained in Eq. (10) from the first t_c -value given previously, which may disfavour a posteriori the second choice of t_c -values, though we do not have a strong argument favouring one with respect to the other⁶. Therefore, we take as a conservative value the largest range spanned by the two sets of results, namely:

$$\begin{aligned}f_\pi &= (86.8 \pm 7.1) \text{ MeV} , \\ \Delta m &\simeq (4.1 \pm 0.9) \text{ MeV} , \\ L_{10} &\simeq -(5.8 \pm 0.2) \times 10^{-3} ,\end{aligned}\quad (12)$$

which we found to be quite satisfactory in the chiral limit. The previous tests are very useful, as they will allow us to gauge the confidence level of the next predictions.

4. Soft pion and kaon reductions of $\langle(\pi\pi)_{I=2}|\mathcal{Q}_{7,8}^{3/2}|K^0\rangle$ to vacuum condensates

We shall consider here the kaon electroweak penguin matrix elements:

$$\langle\mathcal{Q}_{7,8}^{3/2}\rangle_{2\pi} \equiv \langle(\pi\pi)_{I=2}|\mathcal{Q}_{7,8}^{3/2}|K^0\rangle ,\quad (13)$$

defined as:

$$\begin{aligned}\mathcal{Q}_7 &\equiv \frac{3}{2}(\bar{s}d)_{V-A} \sum_{u,d,s} e_\psi (\bar{\psi}\psi)_{V+A} , \\ \mathcal{Q}_8 &\equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{u,d,s} e_\psi (\bar{\psi}_\beta \psi_\alpha)_{V+A} ,\end{aligned}\quad (14)$$

where α, β are colour indices; e_ψ denotes the electric charges. In the chiral limit $m_{u,d,s} \sim m_\pi^2 \simeq m_K^2 = 0$, one can use soft pion and kaon techniques in order to relate the previous amplitude

⁶Approach based on $1/N_c$ expansion and a saturation of the spectral function by the lowest state within a narrow width approximation (NWA) favours the former value of t_c given in Eq. (6) [16].

to the four-quark vacuum condensates [13] (see also [16]):

$$\begin{aligned}\langle\mathcal{Q}_7^{3/2}\rangle_{2\pi} &\simeq -\frac{4}{f_\pi^3}\langle\mathcal{O}_7^{3/2}\rangle , \\ \langle\mathcal{Q}_8^{3/2}\rangle_{2\pi} &\simeq -\frac{4}{f_\pi^3}\left\{\frac{1}{3}\langle\mathcal{O}_7^{3/2}\rangle + \frac{1}{2}\langle\mathcal{O}_8^{3/2}\rangle\right\} ,\end{aligned}\quad (15)$$

where we use the shorthand notations: $\langle 0|\mathcal{O}_{7,8}^{3/2}|0\rangle \equiv \langle\mathcal{O}_{7,8}^{3/2}\rangle$, and $f_\pi = (92.42 \pm 0.26) \text{ MeV}$ ⁷. Here:

$$\begin{aligned}\mathcal{O}_7^{3/2} &= \sum_{u,d,s} \bar{\psi}\gamma_\mu \frac{\tau_3}{2} \psi \bar{\psi}\gamma_\mu \frac{\tau_3}{2} \psi \\ &\quad - \bar{\psi}\gamma_\mu \gamma_5 \frac{\tau_3}{2} \psi \bar{\psi}\gamma_\mu \gamma_5 \frac{\tau_3}{2} \psi , \\ \mathcal{O}_8^{3/2} &= \sum_{u,d,s} \bar{\psi}\gamma_\mu \lambda_a \frac{\tau_3}{2} \psi \bar{\psi}\gamma_\mu \lambda_a \frac{\tau_3}{2} \psi \\ &\quad - \bar{\psi}\gamma_\mu \gamma_5 \lambda_a \frac{\tau_3}{2} \psi \bar{\psi}\gamma_\mu \gamma_5 \lambda_a \frac{\tau_3}{2} \psi ,\end{aligned}\quad (16)$$

where τ_3 and λ_a are flavour and colour matrices. Using further pion and kaon reductions in the chiral limit, one can relate this matrix element to the B -parameters:

$$\begin{aligned}B_7^{3/2} &\simeq \frac{3}{4} \frac{(m_u + m_d)}{m_\pi^2} \frac{(m_u + m_s)}{m_K^2} \frac{1}{f_\pi} \langle\mathcal{Q}_7^{3/2}\rangle_{2\pi} \\ B_8^{3/2} &\simeq \frac{1}{4} \frac{(m_u + m_d)}{m_\pi^2} \frac{(m_u + m_s)}{m_K^2} \frac{1}{f_\pi} \langle\mathcal{Q}_8^{3/2}\rangle_{2\pi}\end{aligned}\quad (17)$$

where all QCD quantities will be evaluated in the \overline{MS} -scheme and at the scale M_τ .

5. The $\langle\mathcal{O}_{7,8}^{3/2}\rangle$ vacuum condensates from DMO-like sum rules in the chiral limit

In previous papers [13,16], the vacuum condensates $\langle\mathcal{O}_{7,8}^{3/2}\rangle$ have been extracted using Das-Mathur-Okubo(DMO)- and Weinberg-like sum rules based on the difference of the vector and axial-vector spectral functions $\rho_{V,A}$ of the $I = 1$ component of the neutral current:

$$\begin{aligned}2\pi\langle\alpha_s\mathcal{O}_8^{3/2}\rangle &= \int_0^\infty ds s^2 \frac{\mu^2}{s + \mu^2} (\rho_V - \rho_A) , \\ \frac{16\pi^2}{3}\langle\mathcal{O}_7^{3/2}\rangle &= \int_0^\infty ds s^2 \times \\ &\quad \log\left(\frac{s + \mu^2}{s}\right) (\rho_V - \rho_A) ,\end{aligned}\quad (18)$$

where μ is the subtraction point. Due to the quadratic divergence of the integrand, the previous sum rules are expected to be sensitive to the high energy tails of the spectral functions

⁷In the chiral limit f_π would be about 84 MeV. However, it is not clear to us what value of f_π should be used here, so we shall leave it as a free parameter which the reader can fix at his convenience.

where the present ALEPH/OPAL data from τ -decay [2,3] are inaccurate. This inaccuracy can a priori affect the estimate of the four-quark vacuum condensates. On the other hand, the explicit μ -dependence of the analysis can also induce another uncertainty. En passant, we check below the effects of these two parameters t_c and μ . After evaluating the spectral integrals, we obtain at $\mu = 2$ GeV and for our previous values of t_c in Eq. (6), the values (in units of 10^{-3} GeV⁶) using the cut-off momentum scheme (c.o):

$$\begin{aligned}\alpha_s \langle \mathcal{O}_8^{3/2} \rangle_{c.o} &\simeq -(0.69 \pm 0.06) , \\ \langle \mathcal{O}_7^{3/2} \rangle_{c.o} &\simeq -(0.11 \pm 0.01) ,\end{aligned}\quad (19)$$

where the errors come mainly from the small changes of t_c -values. If instead, we use the second set of values of t_c in Eq. (5), we obtain by setting $\mu=2$ GeV:

$$\begin{aligned}\alpha_s \langle \mathcal{O}_8^{3/2} \rangle_{c.o} &\simeq -(0.6 \pm 0.3) , \\ \langle \mathcal{O}_7^{3/2} \rangle_{c.o} &\simeq -(0.10 \pm 0.03) ,\end{aligned}\quad (20)$$

which is consistent with the one in Eq. (19), but with larger errors as expected. We have also checked that both $\langle \mathcal{O}_8^{3/2} \rangle$ and $\langle \mathcal{O}_7^{3/2} \rangle$ increase in absolute value when μ increases where a stronger change is obtained for $\langle \mathcal{O}_7^{3/2} \rangle$, a feature which has been already noticed in [16]. In order to give a more conservative estimate, we consider as our final value the largest range spanned by our results from the two different sets of t_c -values. This corresponds to the one in Eq. (20) which is the less accurate prediction. We shall use the relation between the momentum cut-off (c.o) and \overline{MS} -schemes given in [13]:

$$\begin{aligned}\langle \mathcal{O}_7^{3/2} \rangle_{\overline{MS}} &\simeq \langle \mathcal{O}_7^{3/2} \rangle_{c.o} \\ &\quad + \frac{3}{8} a_s \left(\frac{3}{2} + 2d_s \right) \langle \mathcal{O}_8^{3/2} \rangle \\ \langle \mathcal{O}_8^{3/2} \rangle_{\overline{MS}} &\simeq \left(1 - \frac{119}{24} a_s \pm \left(\frac{119}{24} a_s \right)^2 \right) \\ &\quad \times \langle \mathcal{O}_8^{3/2} \rangle_{c.o} - a_s \langle \mathcal{O}_7^{3/2} \rangle ,\end{aligned}\quad (21)$$

where $d_s = -5/6$ (resp $1/6$) in the so-called Naïve Dimensional Regularization NDR (resp. t'Hooft-Veltmann HV) schemes⁸; $a_s \equiv \alpha_s/\pi$. One can notice that the a_s coefficient is large in the 2nd relation (50% correction), and the situation is worse because of the relative minus sign between the two contributions. Therefore, we have added a rough estimate of the a_s^2 corrections based on the naïve growth of the PT series, which here gives 50% corrections of the sum of the two first terms. For a consistency of the whole approach, we shall use the value of α_s obtained from

⁸The two schemes differ by the treatment of the γ_5 matrix.

τ -decay, which is [2,3]:

$$\begin{aligned}\alpha_s(M_\tau)|_{exp} &= 0.341 \pm 0.05 \implies \\ \alpha_s(2 \text{ GeV}) &\simeq 0.321 \pm 0.05 .\end{aligned}\quad (22)$$

Then, we deduce (in units of 10^{-4} GeV⁶) at 2 GeV:

$$\begin{aligned}\langle \mathcal{O}_7^{3/2} \rangle_{\overline{MS}} &\simeq -(0.7 \pm 0.2) , \\ \langle \mathcal{O}_8^{3/2} \rangle_{\overline{MS}} &\simeq -(9.1 \pm 6.4) ,\end{aligned}\quad (23)$$

where the large error in $\langle \mathcal{O}_8^{3/2} \rangle$ comes from the estimate of the a_s^2 corrections appearing in Eq. (21). In terms of the B factor and with the previous value of the light quark masses in Eq. (??), this result, at $\mu = 2$ GeV, can be translated into:

$$\begin{aligned}B_7^{3/2} &\simeq (0.7 \pm 0.2) \left(\frac{m_s(2) [\text{MeV}]}{119} \right)^2 k^4 , \\ B_8^{3/2} &\simeq (2.5 \pm 1.3) \left(\frac{m_s(2) [\text{MeV}]}{119} \right)^2 k^4 .\end{aligned}\quad (24)$$

where:

$$k \equiv \frac{92.4}{f_\pi [\text{MeV}]} .\quad (25)$$

- Our results in Eqs. (23) compare quite well with the ones obtained by [13] in the \overline{MS} -scheme (in units of 10^{-4} GeV⁶) at 2 GeV:

$$\begin{aligned}\langle \mathcal{O}_8^{3/2} \rangle_{\overline{MS}} &\simeq -(6.7 \pm 0.9) , \\ \langle \mathcal{O}_7^{3/2} \rangle_{\overline{MS}} &\simeq -(0.70 \pm 0.10) ,\end{aligned}\quad (26)$$

using the same sum rules but presumably a slightly different method for the uses of the data and for the choice of the cut-off in the evaluation of the spectral integral.

- Our errors in the evaluation of the spectral integrals, leading to the values in Eqs. (19) and (20), are mainly due to the slight change of the cut-off value t_c ⁹.
- The error due to the passage into the \overline{MS} -scheme is due mainly to the truncation of the QCD series, and is important (50%) for $\langle \mathcal{O}_8^{3/2} \rangle$ and $B_8^{3/2}$, which is the main source of errors in our estimate.
- As noticed earlier, in the analysis of the pion mass-difference, it looks more natural to do the subtraction at t_c . We also found that moving the value of μ can affects the value of $B_{7,8}^{3/2}$.

⁹A slight deviation from such a value affects notably previous predictions as the t_c -stability of the results ($t_c \approx 2$ GeV²) does not coincide with the one required by the 2nd Weinberg sum rules. At the stability point the predictions are about a factor 3 higher than the one obtained previously.

For the above reasons, we expect that the results given in [13] for $\langle \mathcal{O}_8^{3/2} \rangle$ though interesting are quite fragile, while the errors quoted there have been presumably underestimated. Therefore, we think that a reconsideration of these results using alternative methods are mandatory.

6. The $\langle \mathcal{O}_{7,8}^{3/2} \rangle$ vacuum condensates from the hadronic tau total decay rates

In the following, we shall not introduce any new sum rule, but, instead, we shall exploit known informations from the total τ -decay rate and available results from it, which have not the previous drawbacks. The V - A total τ -decay rate, for the $I = 1$ hadronic component, can be deduced from BNP [1], and reads ¹⁰:

$$R_{\tau,V-A} = \frac{3}{2} |V_{ud}|^2 S_{EW} \sum_{D=2,4,\dots} \delta_{V-A}^{(D)}. \quad (27)$$

$|V_{ud}| = 0.9753 \pm 0.0006$ is the CKM-mixing angle, while $S_{EW} = 1.0194$ is the electroweak corrections [18]. In the following, we shall use the BNP results for $\mathcal{R}_{\tau,V/A}$ in order to deduce $R_{\tau,V-A}$:

- The chiral invariant $D = 2$ term due to a short distance tachyonic gluon mass [19,20] cancels in the V - A combination. Therefore, the $D = 2$ contributions come only from the quark mass terms:

$$M_\tau^2 \delta_{V-A}^{(2)} \simeq 8 \left[1 + \frac{25}{3} a_s(M_\tau) \right] m_u m_d, \quad (28)$$

as can be obtained from the first calculation [6], where $m_u \equiv m_u(M_\tau) \simeq (3.5 \pm 0.4)$ MeV, $m_d \equiv m_d(M_\tau) \simeq (6.3 \pm 0.8)$ MeV [21] are respectively the running coupling and quark masses evaluated at the scale M_τ .

- The dimension-four condensate contribution reads:

$$M_\tau^4 \delta_{V-A}^{(4)} \simeq 32\pi^2 \left(1 + \frac{9}{2} a_s^2 \right) m_\pi^2 f_\pi^2 + \mathcal{O}(m_{u,d}^4), \quad (29)$$

where we have used the $SU(2)$ relation $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle$ and the Gell-Mann-Oakes-Renner PCAC relation:

$$(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle = -2m_\pi^2 f_\pi^2. \quad (30)$$

- By inspecting the structure of the combination of dimension-six condensates entering in $R_{\tau,V/A}$ given by BNP [1], which are renormalization group invariants, and using

a $SU(2)$ isospin rotation which relates the charged and neutral (axial)-vector currents, the $D = 6$ contribution reads:

$$M_\tau^6 \delta_{V-A}^{(6)} = -2 \times 48\pi^4 a_s \left[\left[1 + \frac{235}{48} a_s \pm \left(\frac{235}{48} a_s \right)^2 - \frac{\lambda^2}{M_\tau^2} \right] \langle \mathcal{O}_8^{3/2} \rangle + a_s \langle \mathcal{O}_7^{3/2} \rangle \right], \quad (31)$$

where the overall factor 2 in front expresses the different normalization between the neutral isovector and charged currents used respectively in [13] and [1], whilst all quantities are evaluated at the scale $\mu = M_\tau$. The last two terms in the Wilson coefficients of $\langle \mathcal{O}_8^{3/2} \rangle$ are new: the first term is an estimate of the NNLO term by assuming a naïve geometric growth of the a_s series; the second one is the effect of a tachyonic gluon mass introduced in [20], which takes into account the resummation of the QCD asymptotic series, with: $a_s \lambda^2 \simeq -0.06$ GeV² ¹¹. Using the values of $\alpha_s(M_\tau)$ given previously, the corresponding QCD series behaves quite well as:

$$\text{Coef. } \langle \mathcal{O}_8^{3/2} \rangle \simeq 1 + (0.53 \pm 0.08) \pm 0.28 + 0.18, \quad (32)$$

where the first error comes from the one of α_s , while the second one is due to the unknown a_s^2 -term, which introduces an uncertainty of 16% for the whole series. The last term is due to the tachyonic gluon mass. This leads to the numerical value:

$$M_\tau^6 \delta_{V-A}^{(6)} \simeq -(1.015 \pm 0.149) \times 10^3 \times \left[(1.71 \pm 0.29) \langle \mathcal{O}_8^{3/2} \rangle + a_s \langle \mathcal{O}_7^{3/2} \rangle \right], \quad (33)$$

- If, one estimates the $D = 8$ contribution using a vacuum saturation assumption, the relevant V - A combination vanishes to leading order of the chiral symmetry breaking terms. Instead, we shall use the combined ALEPH/OPAL [2,3] fit for $\delta_{V/A}^{(8)}$, and deduce:

$$\delta_{V-A}^{(8)}|_{exp} = -(1.58 \pm 0.12) \times 10^{-2}. \quad (34)$$

¹⁰Hereafter we shall work in the \overline{MS} -scheme.

¹¹This contribution may compete with the dimension-8 operators discussed in [22].

We shall also use the combined ALEPH/OPAL data for $R_{\tau,V/A}$, in order to obtain:

$$R_{\tau,V-A}|_{exp} = (5.0 \pm 1.7) \times 10^{-2}, \quad (35)$$

Using the previous informations into the expression of the rate given in Eq. (27), one can deduce:

$$\delta_{V-A}^{(6)} \simeq (4.49 \pm 1.18) \times 10^{-2}. \quad (36)$$

This result is in good agreement with the result obtained by using the ALEPH/OPAL fitted mean value for $\delta_{V/A}^{(6)}$:

$$\delta_{V-A}^{(6)}|_{fit} \simeq (4.80 \pm 0.29) \times 10^{-2}. \quad (37)$$

We shall use as a final result the average of these two determinations, which coincides with the most precise one in Eq. (37). We shall also use the result:

$$\frac{\langle \mathcal{O}_7^{3/2} \rangle}{\langle \mathcal{O}_8^{3/2} \rangle} \simeq \frac{1}{8.3} \left(\text{resp. } \frac{3}{16} \right), \quad (38)$$

where, for the first number we use the value of the ratio of $B_7^{3/2}/B_8^{3/2}$ which is about $0.7 \sim 0.8$ from e.g. lattice calculations quoted in Table 1, and the formulae in Eqs. (15) to (17); for the second number we use the vacuum saturation for the four-quark vacuum condensates [8]. The result in Eq. (38) is also comparable with the estimate of [13] from the sum rules given in Eq.(18). Therefore, at the scale $\mu = M_\tau$, Eqs. (31), (37) and (38) lead, in the \overline{MS} -scheme, to:

$$\langle \mathcal{O}_8^{3/2} \rangle (M_\tau) \simeq -(0.94 \pm 0.21) \times 10^{-3} \text{ GeV}^6, \quad (39)$$

where the main errors come from the estimate of the unknown higher order radiative corrections. It is instructive to compare this result with the one using the vacuum saturation assumption for the four-quark condensate (see e.g. BNP):

$$\begin{aligned} \langle \mathcal{O}_8^{3/2} \rangle|_{v.s.} &\simeq -\frac{32}{18} \langle \bar{u}u \rangle^2 (M_\tau) \\ &\simeq -0.65 \times 10^{-3} \text{ GeV}^6, \end{aligned} \quad (40)$$

which shows a 1σ violation of this assumption. This result is not quite surprising, as analogous deviations from the vacuum saturation have been already observed in other channels [14]. We have used for the estimate of $\langle \bar{\psi}\psi \rangle$ the value of $(m_u + m_d)(M_\tau) \simeq 10 \text{ MeV}$ [21] and the GMOR pion PCAC relation. However, this violation of the vacuum saturation is not quite surprising, as a similar fact has also been observed in other channels [14,2,3], though it also appears that the vacuum saturation gives a quite good approximate value of the ratio of the condensates [14,2,3]. The result in Eq. (39) is comparable with the value $-(.98 \pm 0.26) \times 10^{-3} \text{ GeV}^6$ at $\mu=2 \text{ GeV} \approx M_\tau$ obtained by [13] using a DMO-like sum rule, but,

as discussed previously, the DMO-like sum rule result is very sensitive to the value of μ if one fixes t_c as in Eq. (6) according to the criterion discussed above. Here, the choice $\mu = M_\tau$ is well-defined, and then the result becomes more accurate (as mentioned previously our errors come mainly from the estimated unknown α_s^3 term of the QCD series). Using Eqs. (15) and (38), our previous result in Eq. (39) can be translated into the prediction on the weak matrix elements in the chiral limit and at the scale M_τ (k is defined in Eq. (25)):

$$\langle (\pi\pi)_{I=2} | \mathcal{Q}_8^{3/2} | K^0 \rangle \simeq (2.58 \pm 0.58) \text{ GeV}^3 k^3 \quad (41)$$

normalized to f_π , which avoids the ambiguity on the real value of f_π to be used in a such expression. Our result is higher by about a factor 2 than the quenched lattice result [23]. A resolution of this discrepancy can only be done after the inclusion of chiral corrections in Eqs. (15) to (17), and after the uses of dynamical fermions on the lattice. However, some parts of the chiral corrections in the estimate of the vacuum condensates are already included into the QCD expression of the τ -decay rate and these corrections are negligibly small. We might expect that chiral corrections, which are smooth functions of m_π^2 will not affect strongly the relation in Eqs. (15) to (17), though an evaluation of their exact size is mandatory. Using the previous mean values of the light quark running masses [21], we deduce in the chiral limit and at the scale M_τ :

$$B_8^{3/2} \simeq (1.70 \pm 0.39) \left(\frac{m_s(M_\tau) [\text{MeV}]}{119} \right)^2 k^4, \quad (42)$$

where k is defined in Eq. (25). One should notice that, contrary to the B -factor, the result in Eq. (41) is independent to leading order on value of the light quark masses.

7. Impact of our results on the CP -violation parameter ϵ'/ϵ

One can combine the previous result of B_8 with the value of the B_6 parameter of the QCD penguin diagram [24]:

$$\begin{aligned} \langle \mathcal{Q}_6^{1/2} \rangle_{2\pi} &\equiv \langle (\pi^+ \pi^-)_{I=0} | \mathcal{Q}_6^{1/2} | K^0 \rangle \\ &\simeq - \left[2 \langle \pi^+ | \bar{u} \gamma_5 d | 0 \rangle \langle \pi^- | \bar{s} u | K^0 \rangle + \right. \\ &\quad \left. \langle \pi^+ \pi^- | \bar{d} d + \bar{u} u | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | K^0 \rangle \right] \\ &\simeq -4 \sqrt{\frac{3}{2}} \left(\frac{m_K^2}{m_s + m_d} \right)^2 \times \\ &\quad \sqrt{2} (f_K - f_\pi) B_6^{1/2}(m_c). \end{aligned} \quad (43)$$

We have estimated the $\langle \mathcal{Q}_6^{1/2} \rangle_{2\pi}$ matrix element by relating its 1st term to the $K \rightarrow \pi l \nu_l$ semi-leptonic form factors as usually done (see e.g.

[25]), while the 2nd term has been obtained from the contribution of the $S_2 \equiv (\bar{u}u + \bar{d}d)$ scalar meson having its mass and coupling fixed by QCD spectral sum rules [14,26] and in the scheme where the observed low mass σ meson results from a maximal mixing between the S_2 and the σ_B associated to the gluon component of the trace of the anomaly [27,28,26]¹²:

$$\theta_\mu^\mu = \frac{1}{4}\beta(\alpha_s)G^2 + (1 + \gamma_m(\alpha_s)) \sum_{u,d,s} m_i \bar{\psi}_i \psi_i, \quad (44)$$

where β and γ_m are the β function and mass anomalous dimension. In this way, one obtains at the scale m_c :

$$B_6^{1/2}(m_c) \simeq 3.7 \left(\frac{m_s + m_d}{m_s - m_u} \right)^2 \times \left[(0.65 \pm 0.09) - (0.53 \pm 0.13) \times \left(\frac{(m_s - m_u) [\text{MeV}]}{142.6} \right) \right], \quad (45)$$

which satisfies the double chiral constraint [30]. We have used the running charm quark mass $m_c(m_c) = 1.2 \pm 0.05$ GeV [31,32]. Evaluating the running quark masses at 2 GeV, with the values given in [21], one deduces:

$$\begin{aligned} B_6^{1/2}(2) &\simeq (1.0 \pm 0.4) \text{ for } m_s(2) = 119 \text{ MeV}, \\ &\leq (1.5 \pm 0.4) \text{ for } m_s(2) \geq 90 \text{ MeV}. \end{aligned} \quad (46)$$

The errors added quadratically have been relatively enhanced by the partial cancellations of the two contributions. Therefore, we deduce the combination:

$$\begin{aligned} \mathcal{B}_{68} &\equiv B_6^{3/2} - 0.48 B_8^{3/2} \\ &\simeq (0.3 \pm 0.4) \text{ for } m_s(2) = 119 \text{ MeV}, \\ &\leq (1.0 \pm 0.4) \text{ for } m_s(2) \geq 90 \text{ MeV}, \end{aligned} \quad (47)$$

where we have added the errors quadratically. Using the approximate simplified expression [24]

$$\frac{\epsilon'}{\epsilon} \approx 14.5 \times 10^{-4} \left(\frac{110}{\bar{m}_s(2) [\text{MeV}]} \right)^2 \mathcal{B}_{68}, \quad (48)$$

one can deduce the result in units of 10^{-4} :

$$\begin{aligned} \frac{\epsilon'}{\epsilon} &\simeq (4 \pm 5) \text{ for } m_s(2) = 119 \text{ MeV}, \\ &\leq (22 \pm 9), \text{ for } m_s(2) \geq 90 \text{ MeV}, \end{aligned} \quad (49)$$

where the errors come mainly from \mathcal{B}_{68} (40%). The upper bound agrees quite well with the world average data [33]:

$$\frac{\epsilon'}{\epsilon} \simeq (19.3 \pm 2.4) \times 10^{-4}. \quad (50)$$

¹²Present data appear to favour this scheme [?].

We expect that the failure of the inaccurate estimate for reproducing the data is not naively due to the value of the quark mass, but may indicate the need for other important contributions than the alone $\bar{q}q$ scalar meson S_2 (not the observed σ)-meson which have not been considered so far in the analysis. Among others, a much better understanding of the effects of the gluonium (expected large component of the σ -meson [27,26,28]) in the amplitude, through presumably a new operator needs to be studied.

8. Summary and conclusions

We have explored the $V-A$ component of the hadronic tau decays for predicting non-perturbative QCD parameters. Our main results are summarized as:

- QCD continuum threshold - transition between the low- and high-energy regimes: Eq. (6).
- Low-energy constants L_{10} , $m_{\pi^\pm} - m_{\pi^0}$ and f_π in the chiral limit:
 - Eq. (10) (best)
 - Eq. (12) (conservative).
- Electroweak penguins:
 - Eq. (24): $B_7^{3/2}$,
 - Eq. (42): $B_8^{3/2}$
 - Eq. (41): $\langle (\pi\pi)_{I=2} | \mathcal{Q}_8^{3/2} | K^0 \rangle$.
- ϵ'/ϵ : Eq. (49)

Our results are compared with some other predictions in Table 1. However, as mentioned in the table caption, a direct comparison of these results is not straightforward due to the different schemes and values of the scale where the results have been obtained. In most of the approaches, the values of $B_7^{3/2}$ are in agreement within the errors and are safely in the range $0.5 \sim 1.0$. For $B_8^{3/2}$ the predictions can differ by a factor 2 and cover the range $0.7 \sim 2.1$. There are strong disagreements by a factor 4 for the values of $B_6^{1/2}$ which range from $0.6 \sim 3.0$. We are still far from a good control of these non-perturbative parameters, which do not permit us to give a reliable prediction of the CP violation parameter ϵ'/ϵ . Therefore, no definite bound for new physics effects can be derived at present, before improvements of these ESM predictions.

Table 1

Penguin B -parameters for the $\Delta S = 1$ process from different approaches at $\mu = 2$ GeV. We use the value $m_s(2) = (119 \pm 12)$ MeV from [21], and predictions based on dispersion relations [13,16] have been rescaled according to it. We also use for our results $f_\pi = 92.4$ MeV, but we give in the text their m_s and f_π dependences. Results without any comments on the scheme have been obtained in the $\overline{MS} - NDR$ -scheme. However, at the present accuracy, one cannot differentiate these results from the ones of $\overline{MS} - HV$ -scheme.

Methods	$B_6^{1/2}$	$B_8^{3/2}$	$B_7^{3/2}$	Comments
Lattice [23]	$0.6 \sim 0.8$ unreliable	$0.7 \sim 1.1$	$0.5 \sim 0.8$	Huge NLO at matching[34]
Large N_c [35]	$0.7 \sim 1.3$ $1.5 \sim 1.7$	$0.4 \sim 0.7$ —	$-0.10 \sim 0.04$ —	$\mathcal{O}(p^0/N_c, p^2)$ scheme? $\mathcal{O}(p^2/N_c); m_q = 0$ scheme?
Models				
Chiral QM [36]	$1.2 \sim 1.7$	~ 0.9	$\approx B_8^{3/2}$	$\mu = .8$ GeV rel. with \overline{MS} ?
ENJL+IVB [37]	2.5 ± 0.4	1.4 ± 0.2	0.8 ± 0.1	NLO in $1/N_c$ $m_q = 0$
$L\sigma$ -model [38]	~ 2	~ 1.2	—	Not unique $\mu \approx 1$ GeV; scheme ?
NL σ -model [39]	$1.6 \sim 3.0$	$0.7 \sim 0.9$	—	M_σ : free; $SU(3)_F$ trunc. $\mu \approx 1$ GeV; scheme ?
Dispersive				
Large N_c + LMD +LSD-match.[16]	—	—	0.9 strong μ -dep.	NLO in $1/N_c$,
DMO-like SR [13]	—	1.6 ± 0.4 huge NLO	0.8 ± 0.2	$m_q = 0$ Strong s , μ -dep.
FSI [40]	1.4 ± 0.3	0.7 ± 0.2	—	Debate for fixing the Slope [41]
This work				
DMO-like SR: [13] revisited	—	2.2 ± 1.5 inaccurate	0.7 ± 0.2	$m_q = 0$ Strong s , μ -dep.
τ -like SR	—	—	inaccurate	t_c -changes
R_τ^{V-A}	—	1.7 ± 0.4	—	$m_q = 0$
$S_2 \equiv (\bar{u}u + \bar{d}d)$ from QSSR	1.0 ± 0.4 $\leq 1.5 \pm 0.4$	—	—	\overline{MS} -scheme $m_s(2) \geq 90$ MeV

REFERENCES

1. E. Braaten, S. Narison and A. Pich, *Nucl. Phys.* **B373** (1992) 581.
2. The ALEPH collaboration, R. Barate et al., *Eur. Phys. J C* **4** (1998) 409.
3. The OPAL collaboration, K. Ackerstaff et al., *Eur. Phys. J C* **7** (1999) 571.
4. S. Weinberg, *Phys. Rev. Lett.* **18** (1967) 507.
5. T. Das, V.S. Mathur and S. Okubo, *Phys. Rev. Lett.* **19** (1967) 859;
T. Das et al., *Phys. Rev. Lett.* **18** (1967) 759.
6. E.G. Floratos, S. Narison and E. de Rafael, *Nucl. Phys.* **B155** (1979) 115;
S. Narison and E. de Rafael, *Nucl. Phys.* **B169** (1979) 253.
7. S. Narison, *Z. Phys.* **C14** (1982) 263.
8. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B147** (1979) 385, 448.
9. S. Weinberg, *Physica* **A96** (1979) 327; J. Gasser and H. Leutwyler, *Ann. Phys.* **158** (1984) 142; *Nucl. Phys.* **B250** (1985) 465.
10. E. de Rafael, *Nucl. Phys. (Proc. Suppl.)* **B, A7** (1989) 1 and hep-ph/9502254, Lectures given at the Tasi-school on CP-violation and the limits of the Standard Model, Boulder-Colorado (1994).
11. A. Pich, hep-ph/9502366; A. Manohar, hep-ph/9607484.
12. S. Narison, hep-ph/0004247 (*Nucl. Phys. B*, in press.)
13. J.F. Donoghue and E. Golowich, hep-ph/9911309.
14. For a review and references to original works, see e.g., S. Narison, *QCD spectral sum rules, Lecture Notes in Physics*, **Vol 26** (1989) ed. World Scientific.
15. M. Davier et al., hep-ph/9802447.
16. M. Knecht, S. Peris and E. de Rafael, QCD 99 Euroconference (Montpellier), *Nucl. Phys. (Proc. Suppl.)* **B86** (2000) 1; E. de Rafael, these proceedings and references therein.
17. R.D. Peccei and J. Solà, *Nucl. Phys.* **B281** (1987) 1.
18. W.J. Marciano and A. Sirlin, *Phys. Rev. Lett.* **56** (1986) 22.
19. For a review, see e.g. V.A. Zakharov, *Nucl. Phys. (Proc. Sup.)* **B74** (1999) 392; F.V. Gubarev, M.I. Polikarpov and V.I. Zakharov, hep-th/9812030.
20. K. Chetyrkin, S. Narison and V.I. Zakharov, *Nucl. Phys.* **B550** (1999) 353.
21. For reviews, see e.g., S. Narison, QCD 99 Euroconference (Montpellier), *Nucl. Phys. (Proc. Suppl.)* **B64** (1998) 210 and original references therein; H. Leutwyler, hep-ph/0011049.
22. J.F. Donoghue, these proceedings; V. Cirigliano, J.F. Donoghue and E. Golowich, hep-ph/0007196.
23. For a review, see e.g., G. Martinelli, hep-ph/9910237; R. Gupta, *Nucl. Phys. (Proc. Suppl.)* **B63** (1998) 278.
24. For a review, see e.g., S. Bosch et al., *Nucl. Phys.* **B565** (2000) 3; G. Buchalla, A.J. Buras and M. Lautenbacher, *Rev. Mod. Phys.* **68** (1996) 1125.
25. A.I. Vainshtein, V.I. Zakharov and M.A. Shifman, *Sov. Phys. JETP* **45**(4) (1977) 670; V.I. Zakharov (private communication).
26. S. Narison, plenary talk given at Hadron 99, *Nucl. Phys.* **A675** (2000) 54c; *ibid*, *Nucl. Phys.* **B509** (1998) 312; *ibid*, *Nucl. Phys. (Proc. Suppl.)* **B64** (1998) 210.
27. S. Narison and G. Veneziano, *Int. J. Mod. Phys* **A4, 11** (1989) 2751.
28. A. Bramon and S. Narison, *Mod. Phys. Lett.* **A4** (1989) 1113.
29. L. Montanet, QCD 99 Euroconference (Montpellier); U. Gastaldi, these proceedings.
30. We thank John Donoghue for mentioning this important point.
31. S. Narison, *Nucl. Phys. (Proc. Suppl.)* **B74** (1999) 304.
32. M. Eidemuller and M. Jamin, these proceedings.
33. The kTeV collaboration, B. Hsiung, QCD99 Euroconference (Montpellier) and these proceedings; The NA48 collaboration, B. Peyaud, QCD99 Euroconference (Montpellier); G. Tatishvili, these proceedings.
34. D. Pekurovsky and G. Kilcup, hep-lat/9812019.
35. T Hambye and P. Soldan, these proceedings.
36. S. Bertolini, J.O. Eeg and M. Fabbri, hep-ph/0002234; *Rev. Mod. Phys.* **72** (2000) 65.
37. J. Prades, these proceedings and private communication.
38. T. Morozumi, C.S. Lim and A.I. Sanda, *Phys. Rev. Lett.* **65** (1990) 404; Y.Y. Keum, U. Nierste and A.I. Sanda, *Phys. Lett.* **B457** (1999) 157.
39. M. Harada et al., hep-ph/9910201.
40. E. Pallante and A. Pich, hep-ph/9911233.
41. A.J. Buras et al., hep-ph/0002116; see however: A. Pich (private communication); G. Mennessier (private communication).